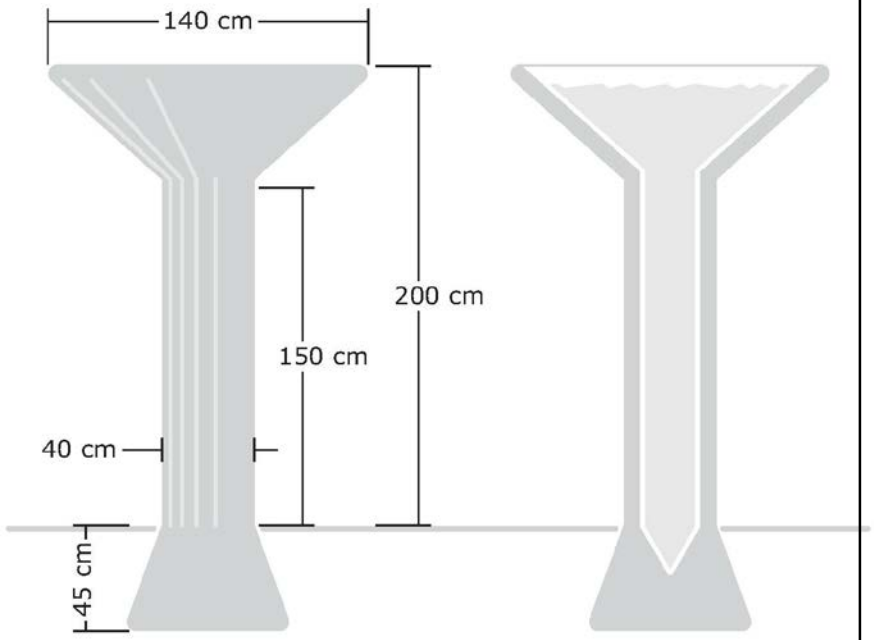
Example Item 4A.1a
Primary Target 4A (Content Domain G-MG), Secondary Target 1O, Tertiary Target 4B, Quaternary Target 4F

A rainwater cistern is shown in the figure.

Estimate the number of liters (*l*) of water the cistern can hold when full.

Enter your estimate in the response box.

[Click here for more information if you need it.[5]]



Interaction: The student can reveal the following list of conversions and formulas:

1 m = 100 cm
1 cm = 10 mm
1 cm^3 = 1 m*l*
1 *l* = 1000 m*l*

Volume of a sphere: $V = \frac{4}{3}\pi r^3$
Volume of a cylinder: $V = \pi r^2 h$
Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Rubric: (1 point) The student enters a reasonable estimate for the amount of water in the cistern (between 340 and 700).

Response Type: Equation/Numeric

[5] Pop-up windows with relevant information are not currently available as in interaction, but it is a planned enhancement for the item authoring tool in 2017.

Example Item 4A.1b
Primary Target 4A (Content Domain NQ), Secondary Target 1C (CCSS N-Q.A), Tertiary Target 4E, Quaternary Target 4D

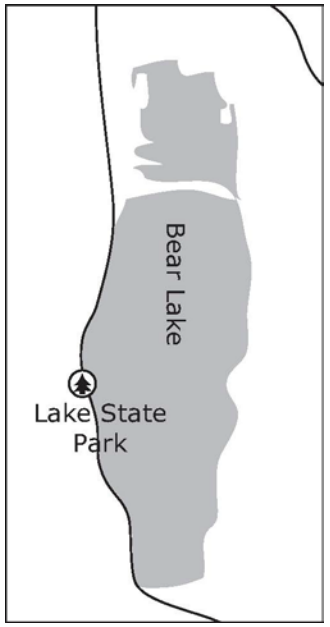
Bear Lake straddles the border between Idaho and Utah. It is about 30 kilometers (km) long and about 11 km wide at the widest spot, as shown in the figure. The water is about 60 meters (m) deep at the deepest spot with an average depth of about 30 m. Use this information to estimate the number of liters (*l*) of water in Bear Lake.

[Click here for more information if you need it.]

Part A:
Enter your estimate in the response box using scientific notation.

Part B:
The surface area of Bear Lake is approximately 280 km^2. Use this information to improve your estimate of the number of liters contained in the lake if you can.

Enter your revised estimate in the response box using scientific notation.



Interaction: The student can reveal the following list of conversions:
1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm
1 cm^3 = 1 m*l*
1 *l* = 1000 m*l*

The student must enter an answer in the first response box before seeing the second response box.

Rubric: (2 points) The student enters a reasonable estimate for Part A for the number of liters in the lake in the first response box (any number between 4.95×10^{12} and 9.9×10^{12}) and enters a more precise estimate for Part B that is closer to 8.4×10^{12}. (1 point) The student enters a reasonable estimate for Part A but does not enter an improved estimate in the second box or the student enters an unreasonable estimate in the first response box and 8.4×10^{12} in the second response box.

High School, Claim 4

Response Type: Equation/Numeric (2 response boxes)

Note: Functionality for this item type does not currently exist. The stimulus can include the information that would be available upon request in the meantime.

Task Model 4A.2

Task Expectations

- The student solves a multi-step problem that requires applying statistics.
- The student identifies needed information and chooses which mathematical structure to employ. The student may
    - ignore irrelevant information,
    - request or conduct research to find missing information, and/or
    - identify constraints that are not explicitly stated.

Example Item 4A.2a

Primary Target 4A (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.B), Tertiary Target 4D, Quaternary Target 4E

This scatterplot shows the income per person (in U.S. dollars) versus the adult literacy rates for 30 countries throughout the world. The adult literacy rate is the percentage of people ages 15 and above who can, with understanding, read and write a short, simple statement about their everyday life.

Part A:
Use the slider to adjust the value for the base for the given exponential model so that it fits the data in the scatterplot.

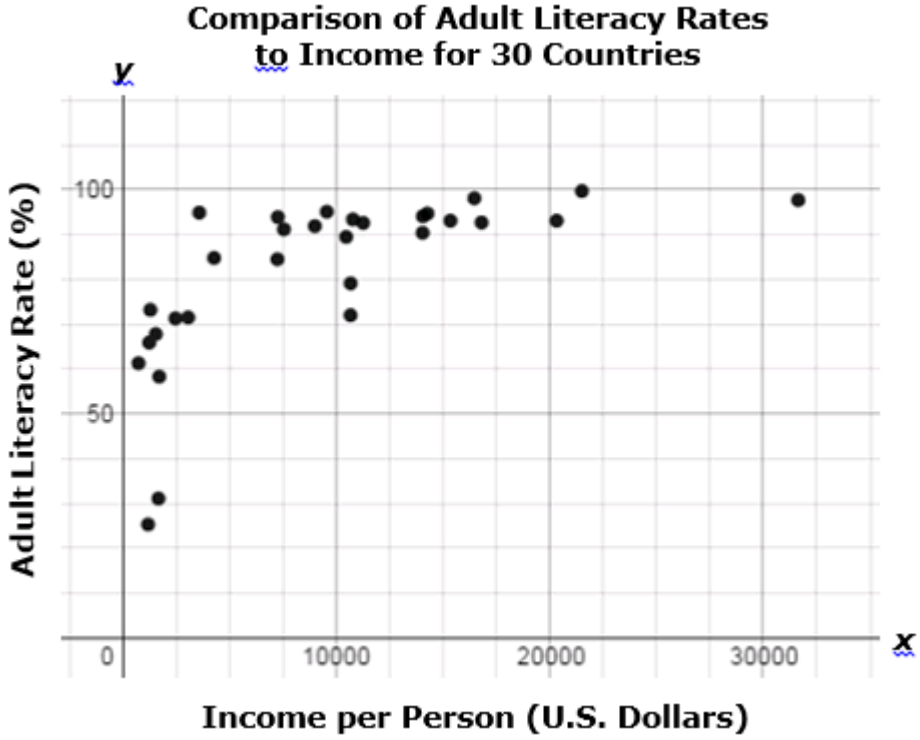
$y = -100 \cdot b^x + 100$

$b = 0.995$

.995 .9999

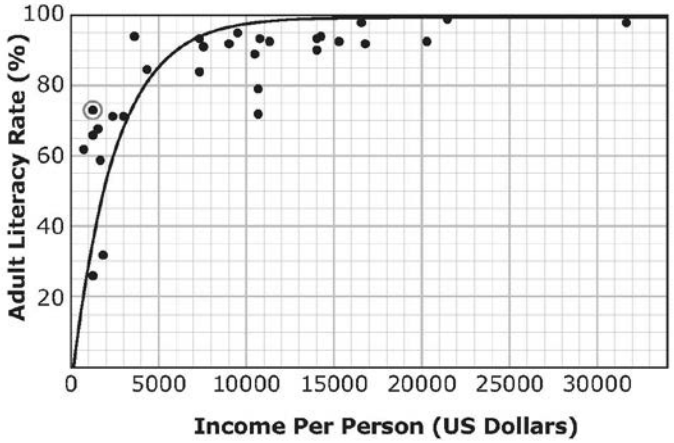
Part B:
Your model predicts the literacy rates in some countries better than others. Which country's literacy rate is least well predicted by your model?

Click on the point in the scatterplot to indicate your choice.



Interaction: The student moves the slider until the graph of the function fits the scatterplot reasonably well (see how this functionality might work here). When the student mouses over a point, the coordinates as well as the name of the country appear. When the student clicks on a point, the name of the country appears and remains on the graph.

Rubric: (1 point) the student adjusts the graph so it fits the scatterplot reasonably well (see exemplar below). At a minimum, at least five of the data points must be above the graph and at least five must be below the graph. The student then clicks on the point that has the greatest vertical distance from their graph.



Response Type: Graphing

Note: The functionality for this item type does not currently exist, but it is planned for a future enhancement to the item-authoring tool.

Data for the scatterplot:

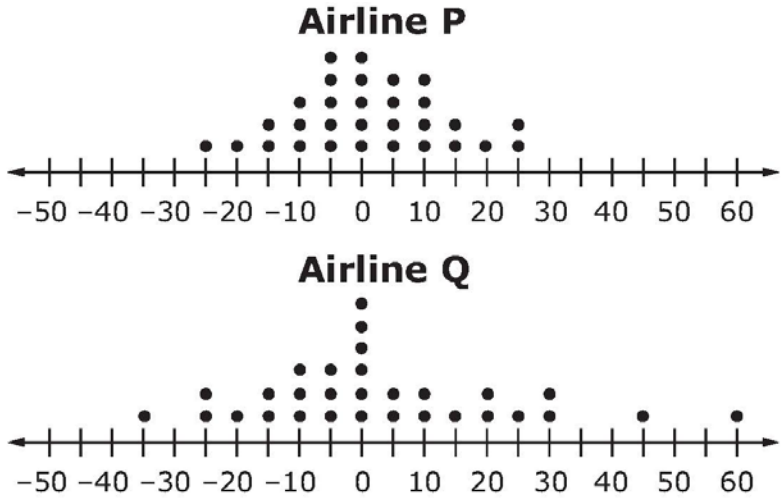
Country	Malawi	Guinea	Rwanda	Mali	Cameroon	Ghana	Honduras	El Salvador	Paraguay	Sri Lanka	Ecuador	China
Income Per Person	737	1,184	1,238	1,669	2,463	3,063	4,277	7,240	7,264	7,546	8,995	9,556
Adult Literacy Rate	61	25	66	31	71	71	85	84	94	91	92	95

Country	Dominican Rep.	Egypt	Colombia	Jordan	Brazil	Panama	Mexico	Malaysia	Russia	Spain	Bahrain	Oman	Singapore
Income Per Person	10,463	10,669	10,782	11,260	14,042	14,050	15,346	20,319	21,501	31,636	40,044	52,799	70,455
Adult Literacy Rate	90	72	93	93	90	94	93	93	100	98	95	87	96

Example Item 4A.2b

Primary Target 4A (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.A), Tertiary Target 4D, Quaternary Target 4B

Two airlines each made 30 flights. The dot plots shown compare how many minutes the actual arrival times were before or after the scheduled arrival times of these flights.



Assuming you want to arrive as close to the scheduled time as possible, from which airline should you buy your ticket? Use all three drop-down menus to respond to this question.

I should choose [Airline P/Airline Q] because the distribution of flights for Airline P has a center that is [greater than/about the same as/less than] the center of the distribution of flights of for Airline Q, and the distribution of flights for Airline P has a spread that is [greater than/about the same as/less than] the spread of the distribution for Airline Q.

- Negative numbers represent the minutes the flight arrived before its scheduled time.
- Positive numbers represent the minutes the flight arrived after its scheduled time.
- Zero indicates the flight arrived at its scheduled time.

Interaction: The response would currently be in a GI box for the drop down functionality to work.

Rubric:
(2 points) The student chooses Airline P, about the same as, and less than.
(1 point) The student chooses Airline P and one of the correct choices for the other two menus.

Response Type: Drop-Down Menu (GI)

Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

General Task Model Expectations for Target 4B

- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

Target 4C: State logical assumptions being used.

General Task Model Expectations for Target 4C

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - identifies information or assumptions needed to solve the problem,
  - researches additional information needed to solve the problem, or
  - provides a reasoned estimate of a quantity needed to solve the problem.

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model generally have either more information than is needed solve the problem (and students identify the relevant information) or not enough information (and students must provide a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks in this model sometimes ask students to choose one of two or more reasonable models, and then to draw the appropriate conclusion base on their choice without evaluating the appropriateness of that choice.
- Tasks for this target may also assess Target 4F.
- Tasks have DOK Level 1 or 2

Task Model 4C.1

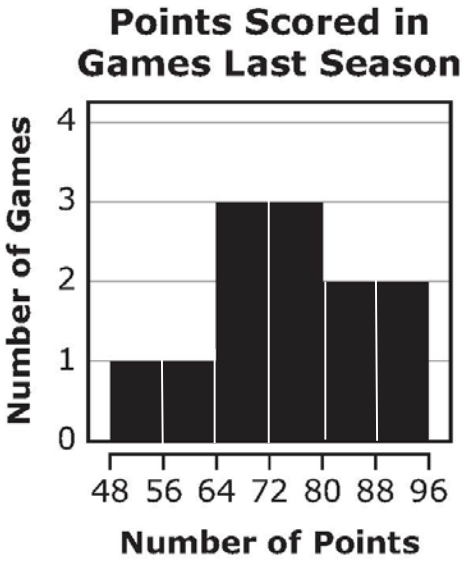
Task Expectations:

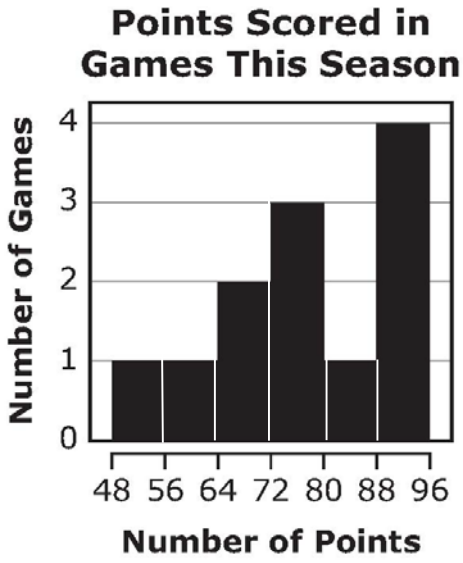
- Students solve problems that involve using stated assumptions, definitions, and previously established results in developing their reasoning.

Example Item 4C.1a
Primary Target 4C (Content Domain S-IC), Secondary Target 1I (CCSS S-ID.A), Tertiary Target 4D

Dirk wants to compare the performance of the school basketball team last year with its performance this year. Decide whether he will use the mean or the median number of points earned in a basketball game to make the comparison. Either measure is acceptable, but you must choose one.

Dirk will use the [mean, median] number of points scored in a game.





Based on the measure of center Dirk will use, which of the following is the best comparison between the performance of the school basketball team last season and this season? You can choose to view the data sets on which the histograms are based if you think it will help you choose.

[Click here to show the data set]

The basketball team performed [better, about the same, worse] this year than last year based on the fact that the [auto-populated with student's choice of *mean* or *median*] is [higher, about the same, lower] this year.

Interaction: The student selects mean or median, which then populates the answer choices. The student can choose to view the underlying dataset—this is not needed if the student chooses mean, but it is if he/she chooses median. The student can change the choice.

Rubric: (1 point) the student selects mean or median, and then selects the answer choice that is best given their selection. If the student selects **mean**, then the correct response choices are **better** and **higher**. If the student selects **median**, then **about the same** should be selected for both choices.

Response Type: Drop-down Menu

Note: Functionality for this item type does not currently exist. The stimulus can include the information that would be available upon request in the meantime. The item can be reconfigured into a hot-spot item by presenting the information on the left side and the choices to complete the statements on the right side, but it wouldn't be fully accessible.

Data Set that displays:

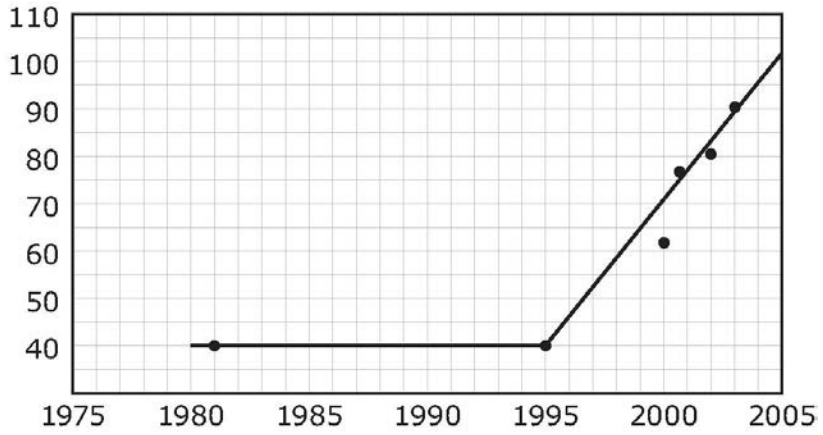
Game #	Scores Last Season	Scores This Season
1	75	73
2	68	71
3	89	94
4	61	67
5	74	72
6	83	85
7	92	94
8	74	62
9	55	54
10	66	75
11	85	90
12	71	90

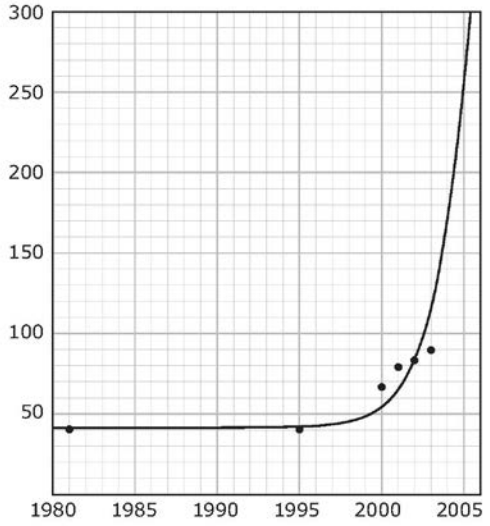
Example Item 4C.1b
Primary Target 4C (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A), Tertiary Target 4D, Quaternary Target 4F

Between 1980 and 1995, there was a single surviving group of Florida panthers that ranged from 30 to 50 individuals in number. In 1995, two females from a closely related species were introduced into this population, and the number of Florida panthers increased to 87 by 2003.

Part A:
Would you model this data with a piece-wise linear function or an exponential function? Select one of these models. Either model is acceptable, but you must choose one. [piece-wise linear, exponential]

Part B:
You chose to model this data with a [auto-populates with student's choice]. This graph shows such a model.





[If the student chose a piece-wise linear function, they see the first graph. If they chose an exponential function, they see second graph.]

What would the approximate population of Florida panthers have been in 2005 according to the model you chose?

Enter your answer in the response box.

Interaction: The student selects a model and then sees the corresponding graph. The student can change his/her choice.

Rubric: (1 point) Student selects piecewise linear or exponential, and then enters the appropriate estimate (between 98 and 100 if piecewise linear, and between 240 and 260 if exponential).

Response Type: Equation/Numeric

Note: Functionality for this item type does not currently exist. The stimulus can include both graphs in the meantime.

Example Item 4C.1c
Primary Target 4C (Content Domain N-NQ), Secondary Target 1C (CCSS NQ.A), Tertiary Target 4F

There is a traffic jam on a highway. A reporter is trying to estimate the number of vehicles involved in the traffic jam. Select all of the information that will help the reporter make a reasonable estimate of the number of vehicles in the traffic jam.

A. The cause of the traffic jam
B. The average length of a vehicle
C. The number of lanes on the highway
D. The average distance between vehicles
E. The average number of people in each vehicle
F. The distance from the beginning to the end of the traffic jam

Rubric: (1 point) Student selects B, C, D, and F.

Response Type: Multiple Choice, multiple correct response

Task Model 4C.2

Task Expectations:

- Students solve problems that involve providing missing information by researching, and/or providing a reasoned estimate.

Example Item 4C.2
Primary Target 4C (Content Domain G-MG), Secondary Target 4F
[Adapted from Illustrative Mathematics 1794]

Eric is using a shovel to clear the snow from his driveway. He moves 8 shovelfuls of snow each minute.
After 60 minutes of hard work, Eric states, "I think I have shoveled more than a ton of snow."

Part A:
Estimate the weight of snow that Eric can move with each shovelful.

If you want to, you can use the table of weights of everyday objects below. A ton is 2000 pounds, and a pound is 16 ounces.

Object	Weight
Basketball	20 ounces
Apple	7 ounces
Bicycle	20 pounds
Car	1.5 tons
Pack of chewing gum	1 ounce

Eric can move [1 ounce/1 pound/10 pounds/0.25 tons] of snow with each shovelful.

Part B:
Use your estimate to decide if Eric's claim is correct or not. [Eric is correct/Eric is not correct]

Rubric: (1 point) Student selects 1 or 10 pounds, and then the corresponding answer to part B (Eric is not correct if one pound was selected, Eric is correct if 10 pounds is selected).

Response type: Drop-down Menu (This can alternately by a hot spot item; but it won't be fully accessible.)

Target 4D: Interpret results in the context of a situation.

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer as well as many problems where students construct an equation or a graph.

General Task Model Expectations for Target 4D

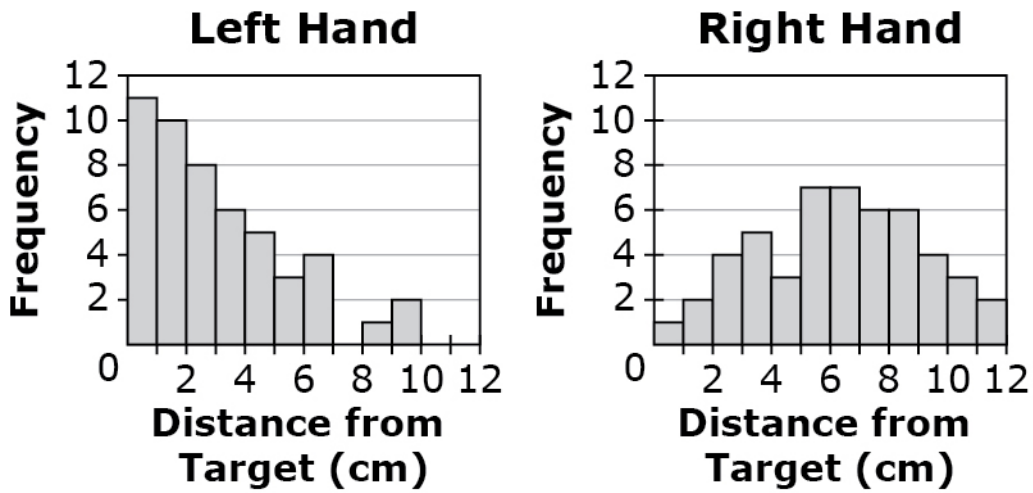
- The student is presented with a problem situation in everyday life, society, or the workplace or a mathematical model of such a situation. The student interprets the solution to the problem in terms of the context, in terms of the model, or compares the results of the model with the real-world data it represents.
    - Item types with a primary alignment to 4D focus on interpreting results in terms of the model or comparing the results of the model with the real-world data it represents.
    - It is not necessary for a student to generate a complete solution for problems with a primary alignment to this target.
- Tasks in Targets 4A, 4C, 4E, and 4F frequently have this target as a tertiary or quaternary alignment because students must interpret their results in terms of the context.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2 or 3.

Task Model 4D.1

- The student is presented with a mathematical model of real-world data.
- The student interprets the solution to the problem in terms of the model or compares the results of the model with the real-world data it represents.

Example Item 4D.1a
Primary Target 4D (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.2)

Lisa was throwing a dart at a target. She threw 50 times with her left hand and 50 times with her right hand.
The histograms show the distance Lisa **missed** the target by each time.



Which statement is an appropriate inference based on the median of each data set?

A. Lisa has better aim with her left hand because the median for her left hand is greater than the median for her right hand.

B. Lisa has better aim with her right hand because the median for her left hand is less than the median for her right hand.

C. Lisa has better aim with her left hand because the median for her left hand is less than the median for her right hand.

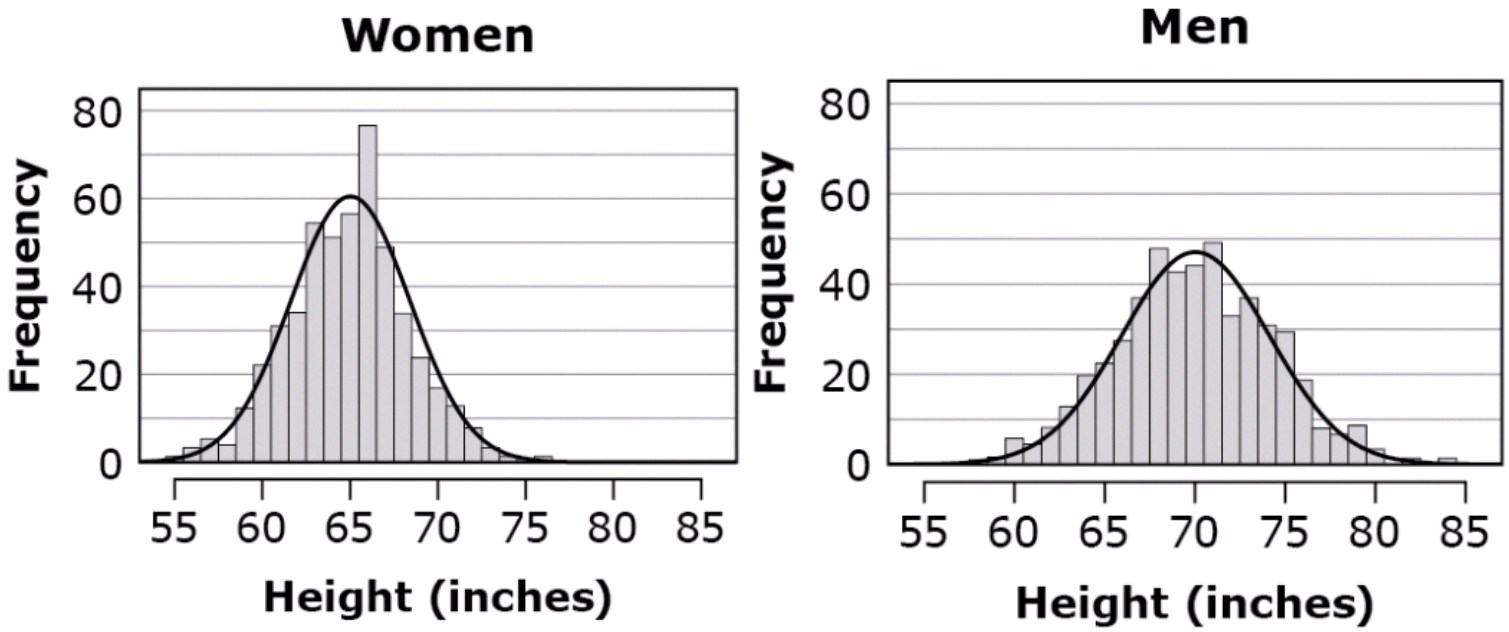
D. Lisa has better aim with her right hand because the median for her left hand is greater than the median for her right hand.

Rubric: (1 point) The student selects the correct option (C).

Response Type: Multiple choice, single correct response

Example Item 4D.1b
Primary Target 4D (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.A)

The distributions of heights of 1000 men and 1000 women selected at random from the population of a large metropolitan area are shown.



Which statement gives an accurate comparison of the heights of men and women shown?

A. The mean height for women is greater than for men and women's heights vary more than men's heights.
B. The mean height for women is greater than for men and women's heights vary less than men's heights.
C. The mean height for women is less than for men and women's heights vary more than men's heights.
D. The mean height for women is less than for men and women's heights vary less than men's heights.

Rubric: (1 point) The student selects the correct comparison statement (D).

Response Type: Multiple Choice, single correct response

Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

General Task Model Expectations for Target 4E

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem),
  - evaluates a partial or complete (possibly incorrect) solution to the problem, or
  - constructs a mathematical model to solve the problem.

  Note: It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task, not just a single step.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2, 3, or 4

Task Model 4E.1

Task Expectations:

- The student constructs a geometric, algebraic, or statistical model for a given context.
- Given data (table of values, scatterplot, etc.) the student will identify the type of function that might best model the situation.
- The student will assess the fit of a particular model being used, including models used in two and three-dimensional geometry.
- May use a simulation that mirrors the functioning of a formula-based online calculator.

Example Item 4E.1a

Primary Target 4E (Content Domain A-CED), Secondary Target 1G (CCSS A-CED.1), Tertiary Target 4B, Quaternary Target 4F

Maia deposited $5500 in a bank account. The money earns interest annually, and the interest is deposited back into her account.

Maia uses an online calculator to determine the amount of money she will have in the account at the end of each year. Follow these steps to use the calculator for each row in the table.

- Select number of years.
- Select "Find Amount."

The amount of money that Maia will have in her account at the end of the selected year, up to 12 years, will appear in the table.

You may use the calculator as many times as you need to help solve the problem.

Years	Money in Bank

Enter an equation that models the amount of money, y, Maia will have in the account at the end of t years.

Rubric: (1 point) The student is able to determine an equation to fit the situation [e.g., $y=5500(1.03)^t$].

Response Type: Equation/Numeric

Example Item 4E.1b:

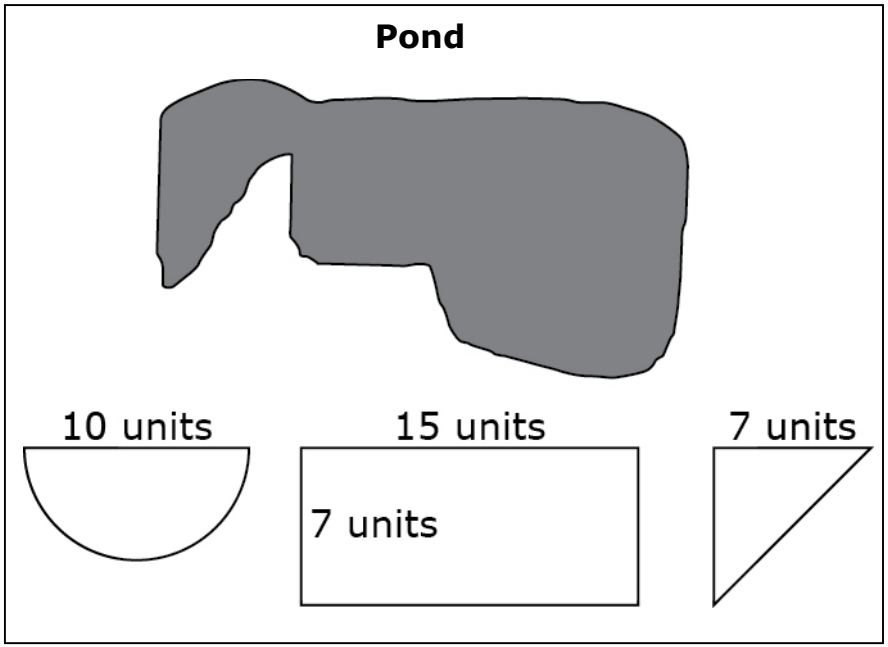
Primary Target 4E (Content Domain G-MG), Secondary Target 1X (G-MG.1), Tertiary Target 4C

A researcher models the area of the surface of a pond using a rectangle, a semi-circle, and an isosceles triangle.

Drag each shape onto the scale diagram of the pond to show how the model fits.

Explain whether the researcher's model will estimate an area greater than, equal to, or less than the actual area of the pond's surface.

Use specific examples and mathematics to support your answer.

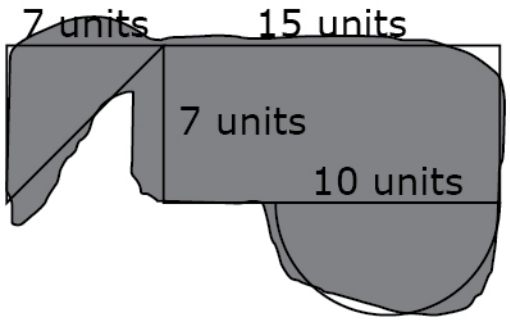


Interaction: The student drags the objects from the palette and places them on top of the pond to approximate the area.

Rubric: (2 points) The student is able to drag the shapes onto the pond in a way to model the best possible fit and make the determination that the pond is slightly larger than the combined areas of the three shapes. The student must supply an explanation that adjusts for the difference in size by either determining the areas of the shapes with specific values or stating how the pond is larger than the combined shapes by a certain portion of one of the shapes (e.g., the triangle needed to be about 2 units longer).

(1 point) The student is able to place the shapes onto the pond in a way to model the best possible fit, but is not able to draw a correct conclusion or support the conclusion.

Exemplar[6]: The area of the pond is slightly greater than the combined area of the three shapes. The semi-circle is the best fit, with a only small amount of the pond extending out the right side, but that is accounted for because of the gap between the pond and the semi-circle at the bottom left side. The rectangle is a good match to the main portion of the pond. However, the triangle is smaller than the remaining portion of the pond. Given the combined area of the three shapes is about 39.3 + 105 +24.5 or 168.8 sq units, I would estimate the pond to be about 175 sq units.



Response Type: Drag and Drop and Short Text[7] (hand scored)

[6] An exemplar response represents only one possible solution. Typically, many other solutions/responses may receive full credit. The full range of acceptable responses is determined during rangefinding and/or scoring validation.
[7] The combination of item response types is currently not available, but it is a planned enhancement to the item authoring tool in 2017.

Task Model 4E.2

Task Expectations:

- Students choose between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.
- The student assesses the fit of a particular model being used.

Example Item 4E.2
Primary Target 4E (Content Domain S-IC), Secondary Target 1I (CCSS S-IC.B), Tertiary Target 4B, Quaternary Target 4C

Lana wrote an article for the school newspaper about the seniors at her school. There are over 1000 students in the senior class and over 4000 students at her school. Lana asked all 40 of the seniors on the swim team whether they intend to go to college, and 22 said yes. She reported in her article that 55% of this year's senior class intends to go to college.

How could Lana improve the accuracy of the information she reported in her article about the seniors at her school? Select **all** that apply.

A. She could report that this only applies to seniors who are swimmers.
B. She should check her arithmetic because she made a mistake computing the percentage.
C. She could ask the guidance counselors what percentage of the senior class went to college last year.
D. There is no need to improve the accuracy of the information she reported because she did everything correctly.
E. She could randomly select 40 students at the all-senior assembly and ask them whether they intend to go to college.

Rubric: (1 point) Student selects all correct answer choices (A and E).

Response Type: Multiple Choice, multiple select response

Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

Task Model 4F.1

Task Model Expectations

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Example Item 4F.1a
Primary Target 4F (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The relationship between Jack's distance from home and the time since he left home is linear, as shown in the table.

Time (hrs)	Distance (mi)
0	7.5
2	17.5
4	27.5

Based on the values in the table, determine whether each statement is true. Select True or False for each statement.

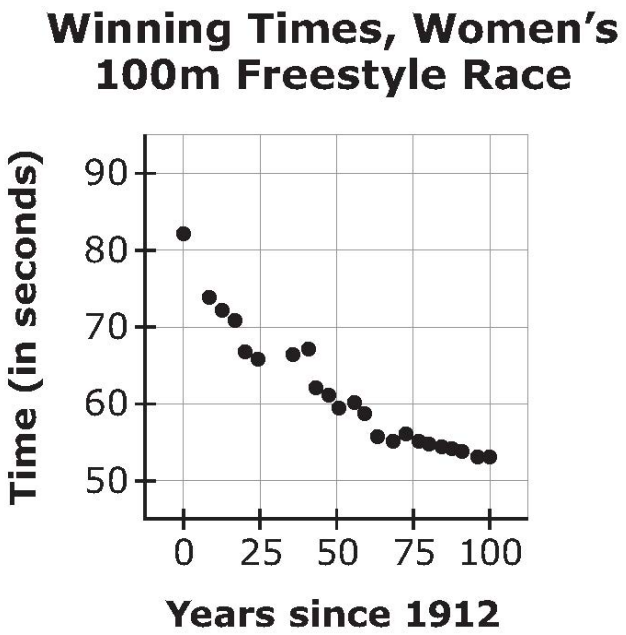
Statement	True	False
Jack's initial distance from home is 7.5 miles.		
Jack's distance increases by 5 miles every 1 hour.		
Jack's distance from home at 3 hours is 23.5 miles.		

Rubric: (1 point) Student determines each statement as being either true or false (TTF).

Example Item 4F.1b

Primary Target 4F (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A), Tertiary Target 4D

The scatterplot shown represents the winning times for the women's 100 meter freestyle race for the Olympic Games between 1912 and 2012.



Part A:
Let x be the year since 1912 and $f(x)$ be the winning time in seconds. Enter either a linear or an exponential function that models the data in the response box.

Part B:
Greta Andersen from Denmark won the race in 1948 with a time of 66.3 seconds. What is the difference between your model's prediction and Greta's actual winning time? Enter your answer in the response box.

Part C:
What does your model predict the winning time will be in the 2016 Olympics? Enter your answer in the response box.

Interaction: The student has access to a Desmos-like graphing calculator with the data already pre-loaded, or some other appropriate graphing or statistical app.

Rubric: (2 points) The student enters an expression for $f(x)$ in the first response box such that $|f(0) - 82| \leq 10$ and $|f(100) - 53| \leq 10$. The student then enters $\pm(f(36)-66.3)$ within a reasonable tolerance in the second response box and enters $f(104)$ in the third response box.

(1 point) The student enters an expression for $f(x)$ in the first response box such that $|f(0) - 82| \leq 10$ and $|f(100) - 53| \leq 10$, or, the student enters an expression that does not meet these criteria but successfully enters either $\pm(f(36)-66.3)$ within a reasonable tolerance in the second response box or enters $f(104)$ in the third response box (or both).

Response Type: Equation/Numeric - label each response type as follows: Part A: $f(x) =$, Part B:, and Part C:.

High School, Claim 4

Data for the scatterplot for example item **4A.1c**:

Year since 1912	0	8	12	16	20	24	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
Time	82.2	73.6	72.4	71	66.8	65.9	66.3	66.8	62	61.2	59.5	60	58.59	55.65	54.79	55.92	54.93	54.65	54.5	53.83	53.84	53.12	53